POLICY FORUM

EDUCATION

More math, less "math war"

A false "equity versus excellence" debate over mathematics curricula has long disrupted education in the United States

By **Alan Schoenfeld**[1] *and* **Phil Daro**[2]

Owing to scientific advances, high-school and college science curricula in the US today barely resemble those of 50 years ago. Most science curricula are "modular": Topics of emerging interest can be inserted as units, without major impact on the broader curriculum. Mathematics instruction, by contrast, is much the same as it was a half century ago—hierarchically organized and inflexible, with each building block taking up a semester or an entire year. Concerns persist around the role of such curricula in creating and perpetuating gaps in math education and subsequent opportunities and outcomes along racial, ethnic, and socioeconomic lines. These tensions lead to a false "equity versus excellence" dichotomy that has fueled recent flare-ups of decades-long "math wars" over curricula and instruction. We suggest that curricular modularity, by which all students can engage the same high-quality mathematics content together, can help resolve this false dichotomy.

K-12 mathematics education in the US is structured in ways that are problematic and do not reflect international trends. The typical math course sequence, algebra I–geometry–algebra II–precalculus–calculus, is much the same as it was more than a half century ago, both in name and in content. This rigid structure makes conflicts almost inevitable. One major criticism has been that the curriculum is "a mile wide and an inch deep" (*1*, *2*). In contrast to the US, in international comparisons, top-scoring nations such as Singapore have coherent national curricula that are established by education ministries and are refined over time. Equally important, teachers in those nations are highly trained and well-regarded professionals who are prepared to make the most of those curricula. Both conditions need to be present for excellence, which is a main reason that, for example, attempts to import Singaporean curricula into the US have not been successful.

[1]School of Education, University of California, Berkeley, Berkeley, CA, USA. [2]Strategic Education Research Partnership (SERP) Institute, Oakland, CA, USA. Email: alans@berkeley.edu

In this policy forum, we address curricula—the focus of the math wars. We do not have the space to address related issues such as teacher preparation, "ambitious instruction," and social contextual factors beyond the classroom [see (*3*) for such a discussion].

EQUITY VERSUS EXCELLENCE

Several long-standing concerns have motivated the math curricula debates (see the first box). Equity advocates, who are concerned about students being diverted into pathways that block opportunities, tend to argue for keeping all students in the pathway that provides access to lucrative careers and academic advancement—historically, the calculus pathway. (Civil rights activist Robert Moses called mathematics education "the civil rights issue of the 21st century.") Excellence advocates, on the other hand, tend to argue that mathematics is by its nature very challenging and that it is inevitable that many students will have difficulty in rigorous math courses, specifically those leading to calculus. From the "excellence" perspective, trying to keep as many students in the calculus-intending pathway for as long as possible necessarily entails diluting the mathematical content, thus harming the students who are needed to advance the STEM enterprise. However, keeping the presently constituted curriculum as the high-priority pathway through secondary mathematics will continue to perpetuate patterns of disenfranchisement that exist now.

Goals for equity can differ depending on student aspirations. Increasing successful participation in the calculus pathway for historically marginalized students improves equity in STEM opportunities. Creating pathways for the broad majority of students who aspire to fields other than STEM improves equity in opportunities for college and careers in those fields. An important goal is to maximize outcomes and opportunities for top STEM students and to maximize outcomes and opportunities for typical students with their varied aspirations, not to serve one group at a cost to the other.

Long-standing concerns

1. Most students are funneled into a single pathway (or "track"), even though career aspirations differ. Almost all students in the US are required to study mathematics yearly through at least 10th grade. Distinctive to the US is that, independent of their intended major [roughly 60% of secondary students do not intend to major in STEM (*14*)], secondary students who plan to attend college typically take mathematics courses from the calculus track through 11th or 12th grade, the latter if they plan to attend selective institutions or major in STEM.
2. Failure and retake rates in high-school mathematics classes have been consistently high, sometimes as high as 50% per year, for decades.
3. Students from underrepresented ethnic and socioeconomic groups experience considerably higher attrition rates (*15*). These students are disproportionately filtered out of mathematics and science. Racial performance gaps in mathematics have remained intractable for decades (*8*). This is a major societal issue, the causes of which include differential access to resources such as up-to-date curricula, qualified teachers, and current technologies, as well as placement systems that assign students of color disproportionately to "remedial" tracks [see (*3*)]. In addition, however, specific mathematics courses, beginning with algebra, are major factors in failure and dropout rates.
4. Historically, course sequences that deviated from the calculus track tended to lead nowhere, creating the perception that any new proposed pathway will lead nowhere. Decades ago, for example, "shop math" and "business math" allowed students to meet mathematics requirements for graduation but did not provide skills that would enhance their employability or enable them to proceed academically beyond high school. Recent discussions about courses in data science have hinged on questions of whether those courses will adequately prepare students for calculus or for college admission (*9*). As increasing numbers of students intend to enroll in college, the path to calculus continues to be seen as the preferred route, despite arguments for the growing importance of data and statistical reasoning.

MATH WARS

Given the inflexibility of the traditional mathematics course sequence, the tension between equity and excellence has seemed intractable. It was a major cause of the math wars that roiled the K-12 educational enterprise for much of the 1990s, stymieing curriculum implementation for more than a decade (*4*, *5*). Those wars started in California, at a time when politicians exploited antiminority sentiment (e.g., with "English language only" initiatives and claims that math was being "dumbed down" to serve minority students) to motivate white voters to come to the polls.

Those math wars raged despite a lack of robust evidence to support either side. Conservatives defended the status quo, which had known problems and compared poorly with the test performance of other nations (*1*); "reformers" of necessity based their proposed changes on small-scale studies, in part, because of the time it takes to design, implement, and test curricula that are intended for large-scale implementation. It took a decade before evaluations convincingly showed that deeper and richer mathematics can serve all students, not just a select subset of them. The first volume of research studies on the new curricula (*6*) indicated that students who received a balanced diet of skills, concepts, and problem-solving suffered no loss of skills and demonstrated sizable gains on tests of conceptual understanding and problem-solving. Change is possible, if undertaken thoughtfully, but it means challenging underlying assumptions about who can do mathematics and how mathematics can be taught.

Although overt hostilities in the math wars had subsided by the time the Common Core State Standards were released in 2010, the underlying tensions remain. These tensions (see the second box) fueled the latest debates and left a status quo in which virtually all of American society continues to be poorly served.

The latest flare-up in California was prompted by an early draft of the 2023 California Mathematics Framework (*7*), which emphasized equity concerns at length. Among early draft recommendations was a new course sequence in data science, a proposed alternative to the standard sequence leading to calculus that could offer students increased options and provide more-equitable access to mathematics and STEM. The proposal generated complaints that the recommended data science pathway jettisoned so much of algebra II that students who took it would be unprepared for first-year calculus in college and would be inadequately prepared to enter a major in data science at some institutions (*8*, *9*). Ironically, then, a course sequence intended to offer viable mathematics-related alternatives to college-intending students had the potential impact of making it more difficult for those students to enter some colleges and earn college-level mathematics credit. Unfortunately, the intensity of the dialogue surrounding these issues rekindled many of the math wars debates, pitching equity advocates against those who argued for excellence.

Ultimately, the revised framework clarified the importance of preparing for calculus for STEM-intending students. Alternative course sequences for students with other aspirations are, again, less clear. This difficulty in providing support for students aspiring to college and careers like nursing, law, and business is testimony to the challenges caused by the rigidity of the math curriculum and the history of alternate courses leading nowhere. Meanwhile, the math wars have been resurrected.

Ongoing challenges

1. Many of the tensions described in the first box remain intractable.
2. The lack of modularity in the curriculum leads to unwanted stability and almost no change—it took 30 years to inject statistics and data analysis into the curriculum, which, in general, does not reflect present mathematical ideas and applications. As a result, all students are confronted with largely archaic and not very interesting or useful curricula. They deserve more up-to-date content, for example, aspects of data science and modeling with functions that use modern computational methods.
3. Early tracking (i.e., deciding to put a student on either the college-preparatory calculus track or the noncalculus track) results in lower-grade-level school personnel making decisions that preclude choice and recovery; late specialization (i.e., delaying placement into alternatives such as statistics or data science until 11th or 12th grade) means having very heterogeneous classes through 10th or 11th grade, with many of the students in those classes neither prepared for the technical demands of current curricula nor interested in it. Specialization must begin at some point; deciding when and how should be a matter of informed pragmatic problem-solving and not shaped by political contests that exploit fears and anger stimulated by math wars.

THREE PRINCIPLES

We believe that the curricular issues discussed here, though not simple, are amenable to progress if approached carefully and with appropriate scientific rigor. Once curricular issues are politicized, careful and nuanced discussion is nearly impossible. That is the risk for mathematics if the math wars are rekindled.

To support curricular improvement in the service of more-powerful mathematics learning, and to avoid perceived tensions between equity and excellence, we need to think outside present curricular boxes. We propose three simple principles for implementing mathematics curricular reform. These principles may seem commonsensical, if not obvious, but they have not been taken up in practice—perhaps because it has been hard to get beyond the idea of semester- or year-long courses as "just the way things are." Tracking has been controversial for decades, with clear evidence that early tracking does little good and much harm (*10*). But if one perceives intact, sequentially organized courses as the only way to arrange a curriculum, then tracking is the only method for offering differentiated access.

Principle 1: Curricula should be designed in ways that preserve as many mathematical pathways and possibilities for all students for as long as possible.

Both STEM- and non-STEM–oriented students should take the same mathematically rich courses together until forced choices between pathways are unavoidable. Each pathway should lead to high-quality opportunities, including college eligibility. In curricular terms, the goal is to redesign the instructional sequence so that all students can profit from the content studied in 8th, 9th, and, possibly, 10th grade. Specialization should emerge as late as possible, but not too late for students to prepare for college majors. Deciding when and how specialization occurs in high school is dense with trade-offs, risks, and benefits. These decisions deserve practical analysis, serious problem-solving, and productive debate.

Principle 2: As the world changes and our understandings grow, mathematics curricula and pedagogy must evolve; modularity (or, at minimum, flexibility) may be a key mechanism.

Tensions over the draft of the California framework were based on the perception that a proposed course in data science would displace algebra II, threatening the straightforward path to calculus. A forced choice between algebra II and data science serves no one: Ideas and content from both areas are likely to be needed in future curricula. [Modeling with functions (i.e., algebra) is at the heart of data science, and the calculus pathway needs more work with data and computation.] Much of the tension between the two courses comes from the perception that such courses are monolithic entities. A modular approach, in which students experience aspects of algebra and data science, could allow students to experience components of both content areas through middle and high school, avoiding false choices between them.

Modular units of topic length are easier to replace or postpone than full courses. A more revolutionary and potentially fruitful idea would be simply to defer teaching less-important but technically demanding operations for as long as possible. Expenditures of time and effort in a course are often determined by the difficulty of a topic, regardless of its importance at the time it is taught. Very important topics of less difficulty get less attention than difficult but unimportant topics. This gives a distorted picture of mathematics to students and teachers. For example, everyone should know the basic ideas of exponential growth and decay and the core concepts of trigonometry. However, few students need to be able to solve complex exponential or logarithmic equations or to manipulate trigonometric identities. Some such work can be done with technological tools, and some can be learned once the students who need it have specialized.

Decomposing and reordering any mathematics course so that the main ideas in it can be introduced early, and complex computations delayed until needed, can provide the flexibility that facilitates modularity. This also provides a mechanism for introducing key areas such as data science and modeling into the curriculum early, when all students can profit from them. In addition, such topics may broaden the appeal of mathematics to more students.

Principle 3: Courses should be suitable for the students who are required to take them, and prior courses should provide robust foundations for later courses.

No rigor should be lost in pursuing the options we have described—the idea is to lay firm foundations for a broad menu of mathematical possibilities while locating difficult technical manipulations in courses after specialization, when they are relevant to student aspirations. The option of taking more courses, whether through pursuing an accelerated sequence, doubling up on courses, or taking summer classes, should be available for those students who wish to pursue more or deeper mathematics.

Designing pathways that align with student aspirations across expanded pools of talent will require collaboration among K-12 systems and higher education to match pathway improvements with college admissions. Improved pathways should fit the aspirations of the roughly 60% of US high school students who are not STEM-intending as well as the approximately 40% who are aimed at STEM-related careers (*7, 11*). Many students aspire to careers in fields such as nursing, construction, business, and law that require postsecondary certification. Which mathematics serves them and their future clients best? They won't be factoring polynomials, but they will be making decisions based on measurement and data.

BUILDING CURRICULA

We do not underestimate the challenges—and costs—of building curricula that rest on these three principles. Initial efforts will not come from textbook publishers who, in the past, have declined this task on the basis of high development costs without guarantees of text adoption. The development of the 1989 National Council of Teachers of Mathematics (NCTM) Standards was underwritten by NCTM, and the National Science Foundation supported the development of textbook series aligned with those standards. The Common Core State Standards were produced under the aegis of the National Governors Association and the Council of Chief State School Officers and underwritten by philanthropic funding. It took a decade to build and test the mathematics curricula that emerged after the publication of the 1989 NCTM Standards. We believe it is possible to redesign curricula along the lines described here, if the political and fiscal wherewithal can be found—perhaps among the groups just mentioned. As the previous reforms show, well-developed curricula designed for the present teaching force do produce results.

Curricula aligned with these principles will meet the needs of all the demographic groups discussed above. Non-STEM–intending students will experience richer but less technically demanding mathematics and have a better sense of the mathematical enterprise. Students from presently underserved groups will reap similar benefits, even if they do not intend to pursue STEM careers, and will still be better supported and prepared for pursuing STEM careers if they decide they are interested in them. With increased modularity, topics that relate to students' lives (e.g., mathematical modeling and data science) may well make mathematics more attractive to both groups of students. In addition, students who aim at careers in mathematics or STEM can pursue those aims without impediment. Curriculum development grounded in principles such as these can avoid the false dichotomy of "equity versus excellence."

There is potential for progress. We know a great deal more about teaching, learning, and equitable and ambitious learning environments than we did even a decade ago, and there are tools for making progress along the lines suggested (*12*, *13*). Yet there is much more to learn about development, about students' identities and how they affect and are affected by schooling, and about the ways in which the social environment plays out in classrooms. We also need to continue addressing basic questions such as: What do we really want students to learn? How do we know when they have? What do teachers need to know to respond productively to students, both as human beings and as learners?

The ideas embodied by the three principles discussed here could be a starting place for breaking out of false dilemmas. Progress can be achieved by coming to agreement on these or other principles and then testing competing ideas. Perhaps funding can be obtained for initial meetings to explore these ideas. What we don't need is more wars. Engaging in academic or political warfare wastes a huge amount of energy that we can ill afford to squander, if we really want to provide richer and more-meaningful learning opportunities for children. ■

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